Three Dimensional Geometry

Question1

Let A be a point having position vector $\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$ and $\mathbf{r} = (\hat{\mathbf{i}} - 3\hat{\mathbf{j}}) + t(\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ be a line. If P is a point on this line and is at a minimum distance from the plane $\mathbf{r}.(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) = 0$, then the equation of the plane through P and perpendicular to AP, is

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Options:

A.

$\mathbf{r} \cdot (-\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 8$

B.

$\mathbf{r} \cdot (\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 4$

C.

$\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 8$

D.

$\mathbf{r} \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}}) = 12$

Answer: A

Solution:

Given, normal vector to the plane is $\mathbf{n} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

And direction vector to the line is $\mathbf{b} = \mathbf{j} - 2\hat{\mathbf{k}}$

Since, $\mathbf{b} \cdot \mathbf{n} = (\hat{\mathbf{j}} - 2\hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$

$= 3 - 10 = -7 \neq 0$

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$\Rightarrow$ line is not perpendicular to the plane Now, $\mathbf{P}(t)=\hat{\mathbf{i}}+(-3+t)\hat{\mathbf{j}}-2t\hat{\mathbf{k}}$

for minimum distance, $\mathbf{p}(t)\cdot\mathbf{n}=0$

$$\begin{aligned}
&\Rightarrow \quad 2+(-9+3t)-10t=0\\
&\Rightarrow \quad -7t-7=0 \Rightarrow t=-1\\
&\qquad \therefore \quad \mathbf{p}=\hat{\mathbf{i}}-4\hat{\mathbf{j}}+2\hat{\mathbf{k}}
\end{aligned}$$

$$\begin{aligned}
\therefore \qquad AP &= (\hat{\mathbf{i}}-4\hat{\mathbf{j}}+2\hat{\mathbf{k}})-(\hat{\mathbf{i}}-3\hat{\mathbf{j}})\\
&= -\hat{\mathbf{j}}+2\hat{\mathbf{k}}
\end{aligned}$$

$\therefore$ Equation of plane

$$\begin{aligned}
&(\mathbf{r}-(\hat{\mathbf{i}}-4\hat{\mathbf{j}}+2\hat{\mathbf{k}}))\cdot(-\hat{\mathbf{j}}+2\hat{\mathbf{k}})=0\\
\Rightarrow \quad &((x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}})-(\hat{\mathbf{i}}-4\hat{\mathbf{j}}+2\hat{\mathbf{k}}))\\
&((-\hat{\mathbf{j}}+2\hat{\mathbf{k}})=0\\
\Rightarrow \quad &((x-1)\hat{\mathbf{i}}+(y+4)\hat{\mathbf{j}}+(z-2\hat{\mathbf{k}})\\
&(-\hat{\mathbf{j}}+2\hat{\mathbf{k}})=0
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow \quad -(y+4)+2(z-2)=0\\
&\Rightarrow \quad -y-4+2z-4=0\\
&\Rightarrow \quad -y+2z-8=0\\
&\Rightarrow \quad -y+2z=8\\
&\Rightarrow \quad \mathbf{r}\cdot(-\hat{\mathbf{j}}+2\hat{\mathbf{k}})=8
\end{aligned}$$

Question2

If L is a line common to the planes $3x+4y+7z=1$, $x-y+z=5$, then the direction ratios of the line L are

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Options:

A.

$(16,0,-1)$

B.

$(11,4,-7)$

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C.

$(2,5,1)$

D.

$(4,-7,11)$

Answer: B

Solution:

Given, equation of planes

$3x+4y+7z=1 \qquad \ldots(i)$

And $\quad x-y+z=5 \qquad \ldots(ii)$

$\therefore$ Direction ratio of normal to the plane (i) is $3,4,7$.

and direction ratio of normal to the plane (ii) is $1,-1,1$

$\therefore$ Direction ratio of L,

$$\begin{aligned}\mathbf{n} &= \mathbf{n}_1 \times \mathbf{n}_2 \\ &= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 4 & 7 \\ 1 & -1 & 1 \end{vmatrix} \\ &= \hat{\mathbf{i}}(4+7)-\hat{\mathbf{j}}(3-7)+\hat{\mathbf{k}}(-3-4) \\ &= 11\hat{\mathbf{i}}+4\hat{\mathbf{j}}-7\hat{\mathbf{k}}\end{aligned}$$

$\therefore$ Direction ratio of L are $11,4,-7$

Question3

If the points $(1,1,\lambda)$ and $(-3,0,1)$ are equidistant from the plane $3x+4y-12z+13=0$, then the values of λ are

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Options:

A.

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$-1, \frac{7}{3}$

B.

$1, \frac{-7}{3}$

C.

$-1, \frac{-7}{3}$

D.

$1, \frac{7}{3}$

Answer: D

Solution:

Given, points $(1, 1, \lambda)$ and $(-3, 0, 1)$ are equidistant from the plane

$$3x + 4y - 12z + 13 = 0$$
$$\therefore \quad \left|\frac{3(1) + 4(1) - 12(\lambda) + 13}{\sqrt{9 + 16 + 144}}\right|$$
$$= \left|\frac{3(-3) + 4(0) - 12(1) + 13}{\sqrt{9 + 16 + 144}}\right|$$
$$\Rightarrow \quad \frac{|20 - 12\lambda|}{\sqrt{169}} = \frac{|-8|}{\sqrt{169}}$$
$$\Rightarrow \quad |20 - 12\lambda| = 8$$

Taking positive sign, we get

$$20 - 12\lambda = 8$$
$$\Rightarrow \quad -12\lambda = -12 \Rightarrow \lambda = 1$$

And taking negative sign, we get

$$\Rightarrow \quad 20 - 12\lambda = -8 \Rightarrow -12\lambda = -28$$
$$\Rightarrow \quad 12\lambda = 28 \Rightarrow \lambda = \frac{7}{3}$$
$$\therefore \quad \lambda = 1, \frac{7}{3}$$

Question4

The shortest distance between the lines

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$\mathbf{r}=(3\hat{\mathbf{i}}-5\hat{\mathbf{j}}+2\hat{\mathbf{k}})+t(4\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}})$ and
$\mathbf{r}=(\hat{\mathbf{i}}+2\hat{\mathbf{j}}-4\hat{\mathbf{k}})+s(6\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2\hat{\mathbf{k}})$ is

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Options:

A.

7

B.

8

C.

9

D.

12

Answer: B

Solution:

Given,

$\mathbf{r}_1=(3\hat{\mathbf{i}}-5\hat{\mathbf{j}}+2\hat{\mathbf{k}})+t(4\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}})$

So, $\mathbf{a}_1=(3\hat{\mathbf{i}}-5\hat{\mathbf{j}}+2\hat{\mathbf{k}})$,

$\mathbf{b}_1=(4\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}})$

And $\mathbf{r}_2=(\hat{\mathbf{i}}+2\hat{\mathbf{j}}-4\hat{\mathbf{k}})+s(6\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2\hat{\mathbf{k}})$

So, $\mathbf{a}_2=(\hat{\mathbf{i}}+2\hat{\mathbf{j}}-4\hat{\mathbf{k}}), \mathbf{b}_2=(6\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2\hat{\mathbf{k}})$

Since, shortest distance,

$d=\frac{|(\mathbf{a}_2-\mathbf{a}_1)\cdot(\mathbf{b}_1\times\mathbf{b}_2)|}{|\mathbf{b}_1\times\mathbf{b}_2|}$

Now, $\mathbf{a}_2-\mathbf{a}_1=\langle 1,2,-4\rangle-\langle 3,-5,2\rangle$

$=\langle 1-3,2-(-5),-4-2\rangle$

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$$=< -2, 7, -6 >$$

$$\mathbf{b}_1 \times \mathbf{b}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4 & 3 & -1 \\ 6 & 3 & -2 \end{vmatrix}$$

$$= (-6 - (-3))\hat{\mathbf{i}} - (-8 - (-6))\hat{\mathbf{j}} + (12 - 18)\hat{\mathbf{k}}$$

$$= -3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$$

$$|\mathbf{b}_1 \times \mathbf{b}_2| = \sqrt{(-3)^2 + 2^2 + (-6)^2}$$

$$= \sqrt{9 + 4 + 36} = \sqrt{49} = 7$$

$$\text{So,} \quad d = \frac{|\langle -2, 7, -6 \rangle \cdot \langle -3, 2, -6 \rangle|}{7}$$

$$= \frac{|6 + 14 + 36|}{7} = \frac{56}{7} = 8$$

Question5

If $A(0, 3, 4), B(1, 5, 6), C(-2, 0, -2)$ are the vertices of a $\triangle ABC$ and the bisector of angle A meets the side BC at D, then $AD =$

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Options:

A.

$\frac{\sqrt{21}}{5}$

B.

$\frac{\sqrt{42}}{10}$

C.

10

D.

4

Answer: B

Solution:

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$$AB = \sqrt{(1-0)^2 + (5-3)^2 + (6-4)^2}$$
$$= \sqrt{1+4+4} = \sqrt{9} = 3$$
$$AC = \sqrt{(-2-0)^2 + (0-3)^2 + (-2-4)^2}$$
$$= \sqrt{4+9+36} = \sqrt{49} = 7$$

Using the angle bisector theorem,

$\frac{BD}{DC} = \frac{AB}{AC} = \frac{3}{7}$

Since, D divides BC in the ratio $3:7$, using the section formula

$$D = \left(\frac{7 \cdot 1 + 3 \cdot (-2)}{7+3}, \frac{7 \cdot 5 + 3 \cdot 0}{7+3}, \frac{7 \cdot 6 + 3 \cdot (-4)}{7+3}\right)$$
$$\Rightarrow \left(\frac{7-6}{10}, \frac{35}{10}, \frac{42-6}{10}\right)$$
$$\Rightarrow \left(\frac{1}{10}, \frac{35}{10}, \frac{36}{10}\right) = (0.1, 35, 3.6)$$

Now,

$$AD = \sqrt{(0.1-0)^2 + (3.5-3)^2 + (3.6-4)^2}$$
$$\Rightarrow \sqrt{(0.1)^2 + (0.5)^2 + (-0.4)^2}$$
$$\Rightarrow \sqrt{0.01 + 0.25 + 0.16} = \sqrt{0.42}$$
$$\Rightarrow \sqrt{\frac{42}{100}} = \frac{\sqrt{42}}{10}$$

Question6

If the direction cosines of two lines satisfy the equation $2l + m - n = 0, l^2 - 2m^2 + n^2 = 0$ and θ is the angle between the lines, then $\cos\theta =$

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Options:

A.

$\frac{1}{5}$

B.

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$\frac{\pi}{4}$

C.

$\frac{2}{3}$

D.

$\frac{\pi}{3}$

Answer: A

Solution:

Given two lines $2l + m - n = 0$

$\Rightarrow \quad n = 2l + m$ and $l^2 - 2m^2 + n^2 = 0$

$\Rightarrow \quad l^2 - 2m^2 + (2l + m)^2 = 0$

$\Rightarrow \quad l^2 - 2m^2 + 4l^2 + 4lm + m^2 = 0$

$\Rightarrow \quad 5l^2 + 4lm - m^2 = 0$

$$\Rightarrow \quad 5\left(\frac{l}{m}\right)^2 + 4\left(\frac{l}{m}\right) - 1 =$$

Let $x = \dfrac{l}{m}, 5x^2 + 4x - 1 = 0$

$$\Rightarrow \quad x = \frac{-4 \pm \sqrt{4^2 - 4(9)(-1)}}{2(9)}$$

$$\Rightarrow \quad \frac{-4 \pm \sqrt{36}}{10} = \frac{-4 \pm 6}{10}$$

$$\therefore \quad x_1 = \frac{2}{10} \text{ and } x_2 = \frac{-10}{10} = -1$$

Case I $\frac{l}{m} = \frac{1}{5}$

$\Rightarrow \quad m = 5l$

So, $n = 2l + m = 2l + 5l = 7l$

Direction cosine (l_1, m_1, n_1) are proportional to $(l, 5l, 7l)$, i.e., $(1, 5, 7)$

Case II $\frac{l}{m} = -1$

$\Rightarrow \quad m = -l$

So, $n = 2l + m = 2l - l = l$

Direction cosine (l_2, m_2, n_2) are proportional to $(l, -l, l)$ i.e., $(1, -1, 1)$

Now,

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$$\cos\theta = \frac{l_1 l_2 + m_1 m_2 + n_1 n_2}{\sqrt{l_1^2 + m_1^2 + n_1^2} \cdot \sqrt{l_2^2 + m_2^2 + n_2^2}}$$
$$\Rightarrow \frac{1 \times 1 + 5 \times (-1) + 7 \times 1}{\sqrt{1^2 + 5^2 + 7^2} \cdot \sqrt{1^2 + (-1)^2 + 1^2}}$$
$$\Rightarrow \frac{1 - 5 + 7}{\sqrt{1 + 25 + 49} \cdot \sqrt{1 + 1 + !}}$$
$$\Rightarrow \frac{3}{\sqrt{75} \cdot \sqrt{3}} = \frac{3}{5 \cdot 3} \Rightarrow \frac{3}{15} = \frac{1}{5}$$
$$\therefore \cos\theta = \frac{1}{5}$$

Question7

If the equation of the plane passing through the points $(2, 1, 2), (1, 2, 1)$ and perpendicular to the plane $2x - y + 2z = 1$ is $ax + by + cz + d = 0$, then $\frac{a+b}{c+d} =$

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Options:

A.

0

B.

1

C.

-1

D.

2

Answer: C

Solution:

Let $P_1 = (2, 1, 2)$ and $P_2 = (1, 2, 1)$

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So, $\mathbf{P}_1\mathbf{P}_2 = P_2 - P_1$

$$\begin{aligned} &\Rightarrow \quad (1,2,1)-(2,1,2) \\ &\Rightarrow \quad (1-2,2-1,1-2 \\ &\Rightarrow \quad (-1,1,-1) \end{aligned}$$

The plane $2x - y + 2z = 1$ has a normal vector $\mathbf{n}_1 = (2, -1, 2)$.

Since, the required plane is perpendicular to $2x - y + 2z = 1$, its normal vector $\mathbf{n} = (a, b, c)$ must be perpendicular to $\mathbf{n}_1$.

So, $\mathbf{n}$ must be perpendicular to $\mathbf{P}_1\mathbf{P}_2$.

$\therefore$ $\mathbf{n}$ is parallel to the cross product of $\mathbf{P}_1\mathbf{P}_2$ and $\mathbf{n}_1$.

$$\mathbf{n} = \mathbf{P}_1\mathbf{P}_2 \times \mathbf{n}_1 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -1 & 1 & -1 \\ 2 & -1 & 2 \end{vmatrix}$$

$$= (2-1)\hat{\mathbf{i}} - (-2+2\hat{\mathbf{j}} + (1-2\hat{\mathbf{k}}$$
$$= \hat{\mathbf{i}} - \hat{\mathbf{k}} = \langle 1, 0, -1 \rangle$$

Now, equation of the point $(2, 1, 2)$ and the normal vector $(1, 0, -1)$ is

$$\begin{aligned} &1(x-2+0(y-1)-1(z-2)=0 \\ &\Rightarrow \quad x-2-z+2=0 \\ &\Rightarrow \quad x-z=0 \end{aligned}$$

Comparing this with $ax + by + cz + d = 0$, we have

$$\begin{aligned} &a=1, b=0, c=-1, d=0 \\ &\therefore \frac{a+b}{c+d} = \frac{1+0}{-1+0} = -1 \end{aligned}$$

Question8

A plane π passing through the points $2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$, $3\hat{\mathbf{i}} + 4\hat{\mathbf{k}}$ is parallel to the vector $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$. If a line joining the points $\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ and $\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ intersects the plane π at the point $a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$, then $a + b + 2c =$

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Options:

A. 31

B. 29

C. 23

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D. 19

Answer: A

Solution:

To find the equation of the plane π that passes through the points $(2, -3, 0)$ and $(3, 0, 4)$, and is parallel to the vector $\langle 2, 3, -4 \rangle$, we start with the general equation of a plane:

$a(x-2) + b(y+3) + c(z-0) = 0.$

Substituting the point $(3, 0, 4)$ into the plane equation gives:

$a(1) + b(3) + c(4) = 0 \quad \text{(i)}.$

Since the plane is parallel to the vector $\langle 2, 3, -4 \rangle$, the plane's normal vector must be orthogonal to $\langle 2, 3, -4 \rangle$. Thus,

$2a + 3b - 4c = 0 \quad \text{(ii)}.$

To find a, b, c such that both conditions are satisfied, compare ratios:

$\frac{a}{-12} = \frac{-b}{-4} = \frac{c}{3}.$

Simplifying gives:

$\frac{a}{-24} = \frac{b}{12} = \frac{c}{-3}.$

From this, solve for proportional constants:

$\frac{a}{8} = \frac{b}{-4} = \frac{c}{1}.$

Choosing $c = 1$, we find $a = 8$ and $b = -4$.

The equation of the plane becomes:

$8(x-2) - 4(y+3) + z = 0,$

or simplified:

$8x - 4y + z - 28 = 0.$

Next, find the line equation that passes through points $(1, 2, 0)$ and $(0, 1, -2)$. The direction vector of the line is $\langle -1, -1, -2 \rangle$.

The parametric equation of the line is:

$\frac{x-1}{-1} = \frac{y-2}{-1} = \frac{z-0}{-2} = \lambda.$

Therefore, the line is:

$P(x, y, z) = (-\lambda + 1, -\lambda + 2, -2\lambda).$

Substitute these into the plane equation:

$8(-\lambda + 1) - 4(-\lambda + 2) + (-2\lambda) - 28 = 0.$

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Simplifying:

$-8\lambda + 8 + 4\lambda - 8 - 2\lambda - 28 = 0,$

$-6\lambda = 28 \quad \Rightarrow \quad \lambda = -\frac{14}{3}.$

Substituting $\lambda = -\frac{14}{3}$ into the line equations gives the intersection point $P(a, b, c) = \left(\frac{17}{3}, \frac{20}{3}, \frac{28}{3}\right).$

Finally, $a + b + 2c$ is:

$a + b + 2c = \frac{17}{3} + \frac{20}{3} + \frac{56}{3} = \frac{93}{3} = 31.$

Question9

$\hat{\mathbf{r}}.(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 5$ and $\hat{\mathbf{r}}.(2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) = 3$ are two planes. A plane π passing through the line of intersection of these two planes, passes through the point $(0, 1, 2)$. If the equation of π is $\hat{\mathbf{r}}.(a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}) = m$, then $\frac{bc}{a^2} =$

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Options:

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 4

D. -4

Answer: D

Solution:

Equation of plane passing through the point of intersection of $x - y + z - 5 = 0$ and $2x + y - z - 3 = 0$ is $x - y + z - 5 + \lambda(2x + y - z - 3) = 0$ π is passing through $(0, 1, 2)$.

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$\Rightarrow \quad 1+\lambda=0$

$\Rightarrow \quad \lambda=-1$

$\Rightarrow x-y+z-5-1(2x+y-z-3)=0$

$-x-2y+2z-2=0$

$x+2y-2z+2=0$

$r \cdot (\hat{\mathbf{i}}+2\hat{\mathbf{j}}-2\hat{\mathbf{k}})=m$

$a=1, b=2, c=-2$

$\because \quad \frac{bc}{a^2}=-\frac{4}{1}$

$\therefore \quad \frac{bc}{a^2}=-4$

Question10

If $A(-2,4,a), B(1,b,3), C(c,0,4)$ and $D(-5,6,1)$ are collinear points, then $a+b+c=$

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Options:

A. 4

B. 8

C. 12

D. -4

Answer: B

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Solution:

$\mathrm{AB} = \mathrm{OB} - \mathrm{OA} = 3\hat{\mathbf{i}} + (b-4)\hat{\mathbf{j}} + (3-a)\hat{\mathbf{k}}$

$\mathbf{CD} = \mathbf{OD} - \mathbf{OC} = (-5-C)\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

AB and **CD** are collinear

$\frac{3}{-5-C} = \frac{b-4}{6} = \frac{3-a}{-3} = \lambda$

$-5 - C = \frac{3}{\lambda}, b - 4 = 6\lambda$ and $3 - a = -3\lambda$

$c = -5 - \frac{3}{\lambda}, b = 6\lambda + 4$ and $a = 3 + 3\lambda$

$a + b + c = -5 - \frac{3}{\lambda} + 6\lambda + 4 + 3 + 3\lambda$

$a + b + c = 2 + 9\lambda - \frac{3}{\lambda}$

By intersection if we put $\lambda = 1$ then, $a + b + c = 2 + 9 - 3 = 8$

Question11

$A(1, -2, 1)$ and $B(2, -1, 2)$ are the end points of a line segment. If $D(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from $C(1, 2, 3)$ to AB, then $\alpha^2 + \beta^2 + \gamma^2 =$

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Options:

A. 18

B. 14

C. 9

D. 27

Answer: A

Solution:

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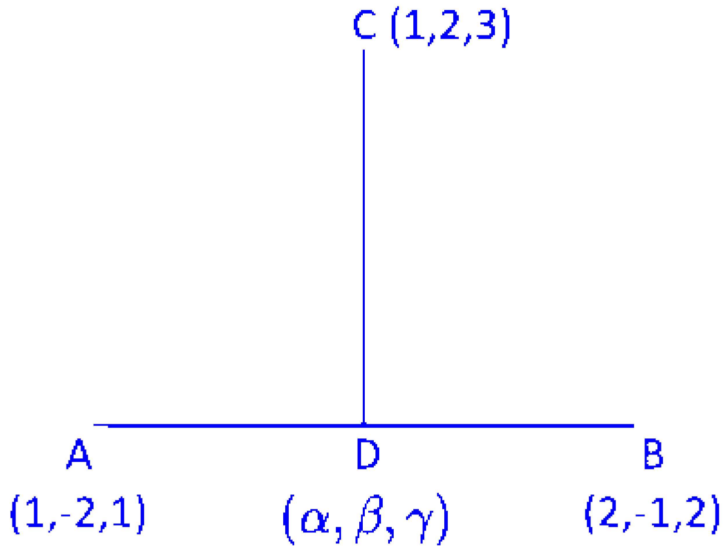

Direction ratio of

AB $\equiv(2-1,-1+2,2-1)=(1,1,1)$

Equation of AB

$\frac{x-1}{1}=\frac{y+2}{1}=\frac{z-1}{1}=D(\text{ say })$

$D \equiv(\lambda+1, \lambda-2, \lambda+1)=D(\alpha, \beta, \gamma)$

$\alpha=\lambda+1, \beta=\lambda-2, \gamma=\lambda+1$

Direction ratio of $CD=\alpha-1, \beta-2, \gamma-3$ $AB \perp CD$

$\alpha-1+\beta-2+\gamma-3=0$

$\alpha+\beta+\gamma=6$

$3 \lambda=6 \Rightarrow \lambda=2$

$\alpha=3, \beta=0, \gamma=3$

$\alpha^2+\beta^2+\gamma^2=9+0+9=18$

Question12

The foot of the perpendicular drawn from the point $(-2,-1,3)$ to a plane π is $(1,0,-2)$. If a, b, c are the intercepts made by the plane π on X, Y, Z-axis respectively, then $3a+b+5c=$

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Options:

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A. 39

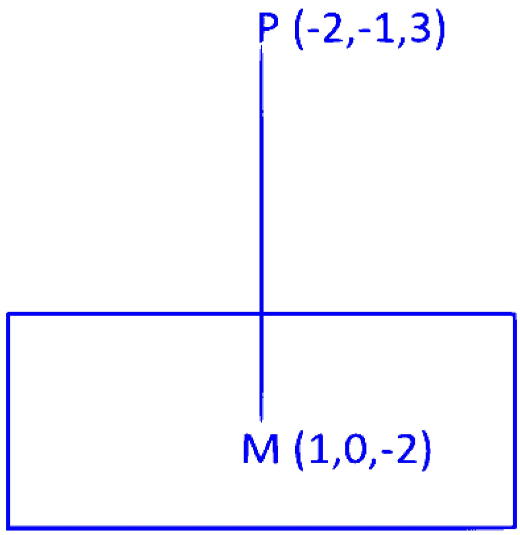
B. 26

C. 13

D. 0

Answer: C

Solution:



Direction ratio of $PM = 3, 1, -5$

Let equation of plane passing through

(x_1, y_1, z_1)

$$3(x - x_1) + 1(y - y_1) - 5(z - z_1) = 0$$

$$3x + y - 5z = 3x_1 + y_1 - 5z_1$$

$M(1, 0, -2)$ lies on the plane

$$13 = 3x_1 + y_1 - 5z_1$$

$$\Rightarrow \quad 3x + y - 5z = 13$$

intercept on axis is

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$$\frac{13}{3}, 13, \frac{-13}{5}$$

$3a + b + 5c = 13 + 13 - 13 = 13$

Question13

n is a unit vector normal to the plane π containing the vectors $\hat{\mathbf{i}} + 3\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$. If this plane π passes through the point $(-3, 7, 1)$ and p is the perpendicular distance from the origin to this plane π, then $\sqrt{p^2 + 5} =$

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. 59

B. 8

C. 64

D. 51

Answer: B

Solution:

To find the unit vector normal to the plane π, we first calculate the cross product of the vectors $\hat{\mathbf{i}} + 3\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$. The cross product is calculated as follows:

$$\mathbf{n} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & 3 \\ 2 & 1 & -1 \end{vmatrix} = (-3)\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

Now, we find the unit vector $\mathbf{n}_{\text{unit}}$ by dividing this normal vector by its magnitude. The magnitude of $-3\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + \hat{\mathbf{k}}$ is:

$$\sqrt{(-3)^2 + 7^2 + 1^2} = \sqrt{9 + 49 + 1} = \sqrt{59}$$

Thus, the unit vector is:

$$\mathbf{n}_{\text{unit}} = \frac{1}{\sqrt{59}}(-3\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

The equation of the plane is given by:

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$-3x+7y+z+d=0$

Since the plane passes through the point $(-3,7,1)$, we substitute these coordinates into the plane's equation:

$-3(-3)+7(7)+1+d=09+49+1+d=0d=-59$

Therefore, the equation of the plane is:

$-3x+7y+z-59=0$

The perpendicular distance p from the origin to the plane can be calculated using the formula for the distance from a point to a plane:

$p=\frac{|0(-3)+0(7)+0(1)-59|}{\sqrt{(-3)^2+7^2+1^2}}=\frac{59}{\sqrt{59}}=\sqrt{59}$

Given p, we calculate:

$\sqrt{p^2+5}=\sqrt{(\sqrt{59})^2+5}=\sqrt{59+5}=\sqrt{64}=8$

Question14

If the harmonic conjugate of $P(2,3,4)$ with respect to the line segment joining the points $A(3,-2,2)$ and $B(6,-17,-4)$ is $Q(\alpha,\beta,\gamma)$, then $\alpha+\beta+\gamma=$

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $-\frac{2}{5}$

B. $-\frac{3}{5}$

C. $\frac{7}{5}$

D. $\frac{8}{5}$

Answer: B

Solution:

To find the harmonic conjugate $Q(\alpha,\beta,\gamma)$ of the point $P(2,3,4)$ with respect to the line segment joining points $A(3,-2,2)$ and $B(6,-17,-4)$, we start by noting that P divides AB in some ratio $\lambda:1$.

First, let's determine this ratio using the x-coordinate:

$2=\frac{6\lambda+3}{\lambda+1}$

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Solving for λ:

$2(\lambda+1)=6\lambda+32\lambda+2=6\lambda+32\lambda-6\lambda=3-2-4\lambda=1\lambda=-\frac{1}{4}$

This indicates that P divides AB in the ratio $-\frac{1}{4}:1$.

For the harmonic conjugate Q, the ratio changes sign, yielding a division ratio of $1:4$ (a harmonized ratio for $-\frac{1}{4}:1$).

We calculate the coordinates of Q using the section formula in the new ratio $1:4$:

$Q=\left(\frac{1\cdot6+4\cdot3}{1+4},\frac{1\cdot(-17)+4\cdot(-2)}{1+4},\frac{1\cdot(-4)+4\cdot2}{1+4}\right)$

Calculating each coordinate:

$x=\frac{6+12}{5}=\frac{18}{5}$

$y=\frac{-17-8}{5}=\frac{-25}{5}=-5$

$z=\frac{-4+8}{5}=\frac{4}{5}$

Thus, $Q=\left(\frac{18}{5},-5,\frac{4}{5}\right)$.

Finally, we find $\alpha+\beta+\gamma$:

$\alpha+\beta+\gamma=\frac{18}{5}-5+\frac{4}{5}=\frac{18}{5}-\frac{25}{5}+\frac{4}{5}=-\frac{3}{5}$

Question15

If L is the line of intersection of two planes $x+2y+2z=15$ and $x-y+z=4$ and the direction ratio of the line L are (a,b,c), then $\frac{(a^2+b^2+c^2)}{b^2}=$

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Options:

A. 14

B. 10

C. 22

D. 26

Answer: D

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Solution:

The line L is the intersection of the two planes given by the equations $x+2y+2z=15$ and $x-y+z=4$. The direction ratios of the line L are determined by the cross product of the normal vectors of these planes.

The normal vector of the first plane is $\langle 1,2,2\rangle$ and for the second plane is $\langle 1,-1,1\rangle$. We calculate the cross product of these two vectors, represented by the determinant:

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 2 & 2 \\ 1 & -1 & 1 \end{vmatrix}$$

This determinant is computed as follows:

$$4\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}}$$

Thus, the direction ratios (a,b,c) of line L are $(4,1,-3)$.

Finally, we need to find the value of:

$$\frac{a^2+b^2+c^2}{b^2}$$

Substituting the values, we get:

$$\frac{4^2+1^2+(-3)^2}{1^2}=\frac{16+1+9}{1}=26$$

Therefore, the result is 26.

Question16

The foot of the perpendicular drawn from $A(1,2,2)$ oril the the plane $x+2y+2z-5=0$ is $B(\alpha,\beta,\gamma)$. If $\pi(x,y,z)$ $=x+2y+2z+5=0$ is a plane, then $-\pi(A):\pi(B)=$

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $15:32$

B. $-7:5$

C. $-15:47$

D. $-27:20$

Answer: B

Solution:

Given that the foot of the perpendicular from $A(1,2,2)$ to the plane $x+2y+2z-5=0$ is $B(\alpha,\beta,\gamma)$, we can begin by determining the direction ratios of the perpendicular.

Start by using the formula for the foot of the perpendicular:

$\frac{\alpha-1}{1}=\frac{\beta-2}{2}=\frac{\gamma-2}{2}=\frac{-(1+4+4-5)}{1+4+4}$

Simplifying, we get:

$\Rightarrow \frac{\alpha-1}{1}=\frac{\beta-2}{2}=\frac{\gamma-2}{2}=\frac{-4}{9}$

Solving these equations, we find:

$\alpha=\frac{5}{9},\quad \beta=\frac{-8}{9}+2=\frac{10}{9},\quad \gamma=\frac{-8}{9}+2=\frac{10}{9}$

Now, calculate $\pi(A)$:

$\pi(A)=1+2\times2+2\times2+5=14$

Next, find $\pi(B)$:

$\pi(B)=\frac{5}{9}+2\left(\frac{10}{9}\right)+2\left(\frac{10}{9}\right)+5=\frac{5}{9}+\frac{20}{9}+\frac{20}{9}+5=10$

Finally, compute the ratio $\frac{-\pi(A)}{\pi(B)}$:

$\frac{-\pi(A)}{\pi(B)}=\frac{-14}{10}=\frac{-7}{5}$

Question17

A plane π_1 passing through the point $3\hat{i}-7\hat{j}+5\hat{k}$ is perpendicular to the vector $\hat{i}+2\hat{j}-2\hat{k}$ and another plane π_2 passing through the point $2\hat{i}+7\hat{k}-8\hat{k}$ is perpendicular to the vector $3\hat{i}+2\hat{j}+6\hat{k}$. If p_1 and p_2 are the perpendicular distances from the origin to the planes π_1 and π_2 respectively, then $p_1-p_2=$

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Options:

A. 1

B. 2

C. 3

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D. 4

Answer: C

Solution:

Equation of plane π_1 is

$1(x-3)+2(y+7)-2(z-5)=0$

$$x+2y-2z-3+14+10=0$$

$$x+2y-2z+21=0 \quad \ldots \text{(i)}$$

Equation of plane π_2 is

$3(x-2)+2(y-7)+6(z+8)=0$

$$3x+2y+6z-6-14+48=0$$

$$3x+2y+6z+28=0 \quad \ldots \text{(ii)}$$

$$P_1=\frac{21}{\sqrt{1+4+4}}=7 \text{ units}$$

$$P_2=\frac{28}{\sqrt{9+4+36}}=4 \text{ units}$$

$P_1-P_2=(7-4)$ units $=3$ units

Question18

$A(2,3,k), B(-1,k,-1)$ and $C(4,-3,2)$ are the vertices of $\triangle ABC$. If $AB=AC$ and $k>0$, then $\triangle ABC$ is

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Options:

A. an equilateral triangle

B. a right-angled isosceles triangle

C. an isosceles triangle but not right angled

D. an obtuse angled isosceles triangle

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Answer: B

Solution:

Given, $A \equiv (2, 3, K), B \equiv (-1, K, -1)$

and $C \equiv (4, -3, 2)$

$AB = AC \Rightarrow |AB|^2 = |AC|^2$

$9 + (K-3)^2 + (1+K)^2 = 4 + 36 + (K-2)^2$

$9 + K^2 + 9 - 6K + 1 + K^2 + 2K$

$40 + K^2 + 4 - 4K$

$2K^2 - 4K + 19 - 40 - K^2 + 4K - 4 = 0$

$K^2 = 44 - 19 = 25 \Rightarrow K = 5$

$A \equiv (2, 3, 5), B \equiv (-1, 5, -1), C \equiv (4, -3, 2)$

$AC = AB = \sqrt{9 + 4 + 36} = 7$

$BC = \sqrt{25 + 64 + 9} = 7\sqrt{2}$

Side are $(7, 7, 7\sqrt{2})$

Question19

If a, b and c are the intercepts made on X, Y, Z-axes respectively by the plane passing through the points $(1, 0, -2), (3, -1, 2)$ and $(0, -3, 4)$, then $3a + 4b + 7c =$

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Options:

A. -5

B. 5

C. -15

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D. 15

Answer: C

Solution:

Equation of plane

$$\begin{vmatrix} x-1 & y & z+2 \\ 3-1 & -1 & 2+2 \\ -1 & -3 & 6 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y & z+2 \\ 2 & -1 & 4 \\ -1 & -3 & 6 \end{vmatrix} = 0$$

$\Rightarrow (x-1)(-6+12) - y(12+4)$

$+(z+2)(-6-1) = 0$

$\Rightarrow 6x - 6 - 16y - 7z - 14 = 0$

$$6x - 16y - 7z = 20$$

x-intercept $= \frac{20}{6} = a$

y-intercept $= \frac{-20}{16} = b$

z-intercept $= \frac{-20}{7} = c$

$3a + 4b + 7c = 10 - 5 - 20 = -15$

Question20

If $\hat{\mathbf{i}} + \hat{\mathbf{j}}, \hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{k}} + \hat{\mathbf{i}}, \hat{\mathbf{i}} - \hat{\mathbf{j}}, \hat{\mathbf{j}} - \hat{\mathbf{k}}$ are the position vectors of the points A, B, C, D, E respectively, then the point of intersection of the line AB and the plane passing through C, D, E is.

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Options:

A. $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$

B. $\frac{1}{2}\hat{\mathbf{i}}+\hat{\mathbf{j}}+\frac{1}{2}\hat{\mathbf{k}}$

C. $\frac{1}{2}\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\Big)$

D. $\frac{1}{2}\hat{i}-\hat{j}+\frac{1}{2}\hat{k}$

Answer: B

Solution:

We have,

$\mathrm{OA}=\hat{\mathbf{i}}+\hat{\mathbf{j}}, \mathrm{OB}=\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$\mathrm{OC}=\hat{\mathbf{k}}+\hat{\mathbf{i}}, \mathrm{OD}=\hat{\mathbf{i}}-\hat{\mathbf{j}}$ and $\mathrm{OE}=\hat{\mathbf{j}}-\hat{\mathbf{k}}$

$\therefore \mathrm{CD}=(\hat{\mathbf{i}}-\hat{\mathbf{j}})-(\hat{\mathbf{k}}+\hat{\mathbf{i}})=-\hat{\mathbf{j}}-\hat{\mathbf{k}}$

$$\mathbf{CE}=(\hat{\mathbf{j}}-\hat{\mathbf{k}})-(\hat{\mathbf{k}}+\hat{\mathbf{i}})=-\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$$
$$\mathbf{DE}=(\hat{\mathbf{j}}-\hat{\mathbf{k}})-(\hat{\mathbf{i}}-\hat{\mathbf{j}})=-\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$$
$$\mathbf{n}=\mathbf{CD}\times\mathbf{CE}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 0 & -1 & -1\\ -1 & 1 & -2\end{vmatrix}=3\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$$

Vector equation of palne,

$$\mathbf{r}\cdot(3\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}})=(\hat{\mathbf{k}}+\hat{\mathbf{i}})\cdot(3\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}})$$
$$\Rightarrow \mathbf{r}\cdot(3\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}})=2 \quad \ldots \text{(i)}$$

Vector equation line,

$$\mathbf{r}=(\hat{\mathbf{i}}+\hat{\mathbf{j}})+t[(\hat{\mathbf{j}}+\hat{\mathbf{k}})-(\hat{\mathbf{i}}+\hat{\mathbf{j}})]$$
$$\Rightarrow \mathbf{r}=(1-t\hat{\mathbf{i}}+\hat{\mathbf{j}}+t\hat{\mathbf{k}} \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$[(1-t)\hat{\mathbf{i}}+\hat{\mathbf{j}}+t\hat{\mathbf{k}}]\cdot(3\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}})=2$$
$$\Rightarrow 3(1-t)+1-t=2 \Rightarrow 3-3t+1-t=2$$
$$\Rightarrow \quad -4t=-2 \Rightarrow t=\frac{1}{2}$$

On putting $t=\frac{1}{2}$ in Eq. (ii), we get

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$$\mathbf{r} = \left(1 - \frac{1}{2}\right)\hat{\mathbf{i}} + \hat{\mathbf{j}} + \frac{1}{2}\hat{\mathbf{k}}$$

$= \frac{1}{2}\hat{\mathbf{i}} + \hat{\mathbf{j}} + \frac{1}{2}\hat{\mathbf{k}}$, which is the required

point of intersection.

Question21

A plane (π) passing through the point $(1, 2, -3)$ is perpendicular to the planes $x + y - z + 4 = 0$ and $2x - y + z + 1 = 0$. If the equation of the plane (π) is $ax + by + cz + 1 = 0$, then $a^2 + b^2 + c^2 =$

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Options:

A. 4

B. 3

C. 2

D. 1

Answer: C

Solution:

Normal vector to the plane

$x + y - z + 4 = 0$ is $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

Normal vector to the plane

$2x - y + z + 1 = 0$ is $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Given planes are perpendicular to the plane (π)

$\therefore$ Normal vector to the plane (π) is given by

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$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & -1 \\ 2 & -1 & 1 \end{vmatrix} = 0\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

$$= -3(0\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$$

Direction Ratio's of $-3(0\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ is ($0, 1, 1$)

Plane (π) is passing through the point ($1, 2, -3$)

$\therefore$ Plane $(\pi) 0(x-1) + 1(y-2) + 1(z+3) = 0$

$\Rightarrow \quad y - 2 + z + 3 = 0$

$\Rightarrow \quad 0x + y + z + 1 = 0$

Given Plane $(\pi) ax + by + cz + 1 = 0$

$\therefore \quad a = 0, b = 1$ and $c = 1$

Now, $a^2 + b^2 + c^2 = 0^2 + 1^2 + 1^2$

$= 0 + 1 + 1 = 2$

Question22

If the ratio of the perpendicular distances of a variable point $P(x, y, z)$ from the X-axis and from the YZ - plane is $2 : 3$, then the equation of the locus of P is

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $4x^2 - 9y^2 - 9z^2 = 0$

B. $9x^2 - 4y^2 - 4z^2 = 0$

C. $4x^2 - 4y^2 - 9z^2 = 0$

D. $9x^2 - 9y^2 - 4z^2 = 0$

Answer: A

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Solution:

We have a variable point $P(x, y, z)$

Distance of P from X-axis

$= \sqrt{0^2 + y^2 + z^2} = \sqrt{y^2 + z^2}$

and distance of P from YZ-plane $= |x|$

Now, $\frac{\sqrt{y^2+z^2}}{|x|} = \frac{2}{3} \quad$ [given]

$\Rightarrow \frac{y^2+z^2}{x^2} = \frac{4}{9} \Rightarrow 4x^2 - 9y^2 - 9z^2 = 0$

which is the equation of locus of P.

Question23

The direction cosines of two lines are connected by the relations $l - m + n = 0$ and $2lm - 3mn + nl = 0$. If θ is the angle between these two lines, then $\cos\theta =$

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $\frac{1}{4}$

B. $\frac{1}{\sqrt{19}}$

C. $\frac{1}{3}$

D. $\frac{1}{3\sqrt{2}}$

Answer: B

Solution:

We have,

$l - m + n = 0 \Rightarrow m = n + l \quad \ldots$ (i)

and $2lm - 3mn + nl = 0$

$\Rightarrow 2l(n + l) - 3(n + l)n + nl = 0$

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$$\Rightarrow 2l^2 - 3n^2 = 0 \quad \text{[from Eq. (i)]}$$

$$(\sqrt{2}l + \sqrt{3}n)(\sqrt{2}l - \sqrt{3}n) = 0$$

$$\Rightarrow \quad n = \frac{\sqrt{2}}{\sqrt{3}}l, \frac{-\sqrt{2}}{\sqrt{3}}l$$

Since, $n = \frac{\sqrt{2}}{\sqrt{3}}l$

$$\Rightarrow \quad m = \left(\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3}}\right)l \quad \text{[from Eq. (i)]}$$

and $n = -\frac{\sqrt{2}}{\sqrt{3}}l \Rightarrow m = \left(\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3}}\right)l$ [from Eq. (i)]

So, $\frac{l}{1} = \frac{n}{\frac{\sqrt{2}}{\sqrt{3}}} = \frac{m}{\left(\frac{\sqrt{2}+\sqrt{3}}{\sqrt{3}}\right)}$ and

$\frac{l}{1} = \frac{n}{-\frac{\sqrt{2}}{\sqrt{3}}} = \frac{m}{\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)}$

Now,

$$\cos\theta = \frac{11 + \frac{\sqrt{2}}{\sqrt{3}} \cdot \left(-\frac{\sqrt{2}}{\sqrt{3}}\right) + \left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}}\right)\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)}{\sqrt{(1)^2 + \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 + \left(\frac{\sqrt{2}+\sqrt{3})^2}{\sqrt{3}}\right)^2}}$$

$$= \frac{1 - \frac{2}{3} + \frac{1}{3}}{\sqrt{\frac{5}{3} + \frac{5+2\sqrt{6}}{3}} \cdot \sqrt{\frac{5}{3} + \frac{5-2\sqrt{6}}{3}}\Big)^2 + \left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)^2}$$

$$= \frac{2/3}{\sqrt{\frac{2}{3}(5+\sqrt{6})} \cdot \sqrt{\frac{2}{3}(5-\sqrt{6})}}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{19}}$$

Question24

A plane π passes through the points $(5, 1, 2), (3, -4, 6)$ and $(7, 0, -1)$. If p is the perpendicular distance from the origin to the plane π and l, m and n are the direction cosines of a normal to the plane π, the $|3l + 2m + 5n| =$

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Options:

A. $3p$

B. $2p$

C. p

D. $\frac{p}{2}$

Answer: C

Solution:

We have,

(c) We have,

$A \equiv (5,1,2), B \equiv (3,-4,6), C \equiv (7,0,-1)$

So, normal vector to plane π is

$\mathbf{n} = \mathbf{AB} \times \mathbf{AC}$

Here, $\mathbf{AB} = -2\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

$AC = 2\hat{\mathbf{i}} - 1\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

So, $\mathbf{n} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -2 & -5 & 4 \\ 2 & -1 & -3 \end{vmatrix}$

$\mathbf{n} = 19\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 12\hat{\mathbf{k}}$

Now, equations of plane through point A with normal vector $(19,2,12)$ is

$19(x-5) + 2(y-1) + 12(z-2) = 0$

So, $P = \frac{|-95-2-24|}{\sqrt{361+4+144}} = \frac{121}{\sqrt{509}}$

Direction cosines of $\mathbf{n}$ is $l = \frac{19}{\sqrt{509}}$,

$m = \frac{2}{\sqrt{509}}, n = \frac{12}{\sqrt{509}}$

Hence,

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$|3l+2m+5n| = \left|\frac{57+4+60}{\sqrt{509}}\right| = \frac{121}{\sqrt{509}} = P$

Question25

If M is the foot of the perpendicular drawn from $P($ -1,2,-1 $)$ to the plane passing through the point $A(3,-2,1)$ and perpendicular to the vector $4\hat{\mathbf{i}}+7\hat{\mathbf{j}}-4\hat{\mathbf{k}}$, then the length of PM is

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $\frac{16}{3}$

B. $\frac{18}{5}$

C. $\frac{22}{9}$

D. $\frac{28}{9}$

Answer: D

Solution:

Now, first to find equation of plane passing through point $A(3,-2,1)$ and perpendicular to the vector $4\hat{\mathbf{i}}+7\hat{\mathbf{j}}-4\hat{\mathbf{k}}$.

$$\text{So, } \mathbf{n}\cdot(4\hat{\mathbf{i}}+7\hat{\mathbf{j}}-4\hat{\mathbf{k}}) = (4\hat{\mathbf{i}}+7\hat{\mathbf{j}}-4\hat{\mathbf{k}})$$
$$\cdot(3\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}})$$
$$\Rightarrow (x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}})\cdot(4\hat{\mathbf{i}}+7\hat{\mathbf{j}}-4\hat{\mathbf{k}})$$
$$=12-14-4=-6$$
$$4x+7y-4z=-6$$
$$\Rightarrow 4x+7y-4z+6=0$$

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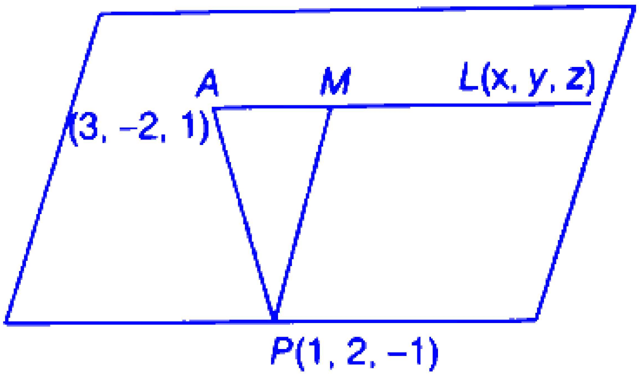

Since, M is the foot of the perpendicular drawn from $P(1,2,-1)$ to the plane $4x+7y-4z+6=0$

So,

$$
\begin{aligned}
PM &= \left|\frac{4(1)+7(2)-4(-1)+6}{\sqrt{16+49+16}}\right| \\
&= \left|\frac{4+14+4+6}{\sqrt{81}}\right| = 28/9
\end{aligned}
$$

Question26

If $A=(1,-1,2), B=(3,4,-2), C=(0,3,2)$ and $D=(3,5,6)$, then the angle between the lines AB and CD is

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. 30°

B. 45°

C. 60°

D. 90°

Answer: D

Solution:

Given, $A=(1,-1,2), B=(3,4,-2), C=(0,3,2)$ and $D=(3,5,6)$

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Now, DR's of the lines **AB** is $(2, 5, -4)$ and DR's of the lines **CD** is $(3, 2, 4)$ Let ' θ ' be the angle between them, we get

$$\cos\theta = \frac{\mathbf{AB}\cdot\mathbf{CD}}{|\mathbf{AB}|\cdot|\mathbf{CD}|}$$
$$= \frac{6+10-16}{\sqrt{2^2+5^2+4^2}\sqrt{3^2+2^2+4^2}}$$
$$\Rightarrow \cos\theta = 0$$
$$\therefore \quad \theta = 90^\circ$$

Question27

Consider the following statements:

Assertion (A) : The direction ratios of a line L_1 are 2,5, 7 and the direction ratios of another line L_2 are $\frac{4}{\sqrt{19}}, \frac{10}{\sqrt{19}}, \frac{14}{\sqrt{19}}$. Then, the lines L_1, L_2 are parallel.

Reason : (R) If the direction ratios of a line L_1 are a_1, b_1, c_1 the direction ratios of a line L_2 are a_2, b_2, c_2 and $a_1a_2 + b_1b_2 + c_1c_2 = 0$, then the lines of L_1, L_2 are parallel.

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Options:

A. (A) and (R) are true, (R) is the correct explanation of (A)

B. (A) and (R) are true, (R) is not the correct explanation of (A)

C. (A) is true, (R) is false

D. (A) is false, (R) is true

Answer: C

Solution:

Assertion (A) DR'S of a line L_1 is 2, 5, 7 and

DR'S of a line L_2 is $\frac{4}{\sqrt{19}}, \frac{10}{\sqrt{19}}, \frac{14}{\sqrt{19}}$

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Thus, $\frac{2}{\frac{4}{\sqrt{19}}} = \frac{5}{\frac{10}{\sqrt{19}}} = \frac{7}{\frac{14}{\sqrt{19}}} = \frac{1}{2}$

Hence, the lines L_1 and L_2 are parallel. Reason (R) If the DR'S of a line L_1 are a_1, b_1, c_1 and the $\mathrm{DR}^{\dagger}\mathrm{S}$ of a line L_2 are a_2, b_2, c_2 and $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$, then the lines are not parallel.

So, Assertion (A) is true but Reason (R) is false.

Question28

A line L is parallel to both the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If line L makes an angle α with the positive direction of X-axis, then $\cos \alpha =$

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $1/\sqrt{3}$

B. $1/\sqrt{2}$

C. $1/2$

D. $\sqrt{3}/2$

Answer: A

Solution:

The given planes are $2x + 3y + z = 1$ and $x + 3y + 2z = 2$

$$\text{Thus, } \hat{\mathbf{n}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{vmatrix} = \hat{\mathbf{i}}(3) + \hat{\mathbf{j}}(-3) + \hat{\mathbf{k}}(3)$$

$$= 3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

$$\text{and} \quad \hat{\mathbf{a}} = \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}}{\sqrt{9+9+9}} = \frac{\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}$$

$$\Rightarrow \quad \mathbf{a} = \frac{1}{\sqrt{3}}\hat{\mathbf{i}} - \frac{1}{\sqrt{3}}\hat{\mathbf{j}} + \frac{1}{\sqrt{3}}\hat{\mathbf{k}}$$

Since, the line ' L ' makes an angle α with the positive direction of X-axis, then

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$\cos\alpha = \frac{1}{\sqrt{3}}$

Question29

If the direction cosines (l, m, n) of two lines are connected by the relations $l + m + n = 0$ and $lm = 0$, then the angle between those lines is

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Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{6}$

Answer: A

Solution:

Given the direction cosines (l, m, n) of two lines, we have the following relationships:

$l + m + n = 0$

$lm = 0$

From the equation $lm = 0$, we infer that either $l = 0$ or $m = 0$.

Case 1: $l = 0$

Substitute $l = 0$ into the first equation: $m + n = 0$.

This gives $m = -n$.

Now, using the condition for direction cosines, $l^2 + m^2 + n^2 = 1$, we substitute $m = -n$ and $l = 0$:

$0^2 + (-n)^2 + n^2 = 1 \implies 2n^2 = 1 \implies n = \frac{1}{\sqrt{2}}$

Therefore, the set of direction cosines is:

$\langle l, m, n\rangle = \left\langle 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle$

Case 2: $m = 0$

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Substitute $m = 0$ into the first equation: $l + n = 0$.

This gives $l = -n$.

Again using $l^2 + m^2 + n^2 = 1$ with $l = -n$ and $m = 0$:

$(-n)^2 + 0^2 + n^2 = 1 \implies 2n^2 = 1 \implies n = \frac{1}{\sqrt{2}}$

Thus, the set of direction cosines is:

$\langle l, m, n \rangle = \left\langle -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\rangle$

Finding the Angle θ Between the Lines

The cosine of the angle θ between the two lines can be calculated using the dot product of their direction cosine vectors. Let us compute it:

$\cos\theta = 0 \times \left(-\frac{1}{\sqrt{2}}\right) + \left(-\frac{1}{\sqrt{2}}\right) \times 0 + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

Thus, the angle θ is:

$\theta = \frac{\pi}{3}$

Question30

The sum of the squares of the perpendicular distances of a point (x, y, z) from the coordinate axes is k times the square of the distance of the point from the origin Then, $k =$

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Options:

A. 2

B. 3

C. 1

D. 4

Answer: A

Solution:

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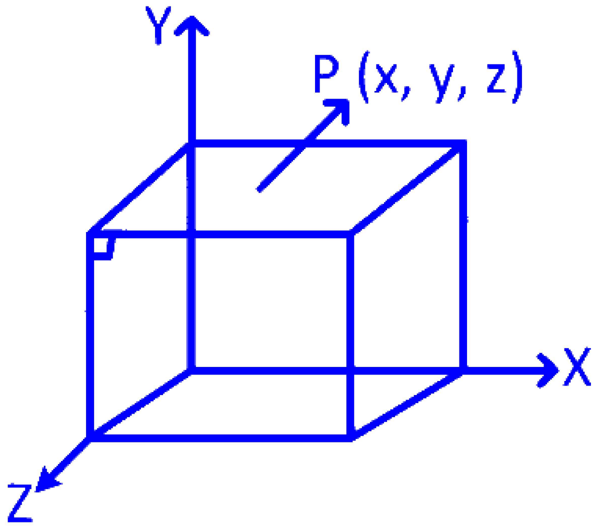

Perpendicular distance on X-axis

$= y^2 + z^2$

Perpendicular distance on Y-axis

$= x^2 + z^2$

Perpendicular distance on Z-axis

$$= x^2 + y^2$$

$\therefore 2\left(x^2 + y^2 + z^2\right) = k\left(x^2 + y^2 + z^2\right)$

Hence, $2 = k$

Question31

Equation of the plane through the mid-point of the line segment joining the points $A(4, 5, -10)$ and $B(-1, 2, 1)$ and perpendicular to AB is

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Options:

A. $10x + 6y - 22z + 135 = 0$

B. $10x + 6y - 22z - 135 = 0$

C. $5x + 3y + 11z = 135$

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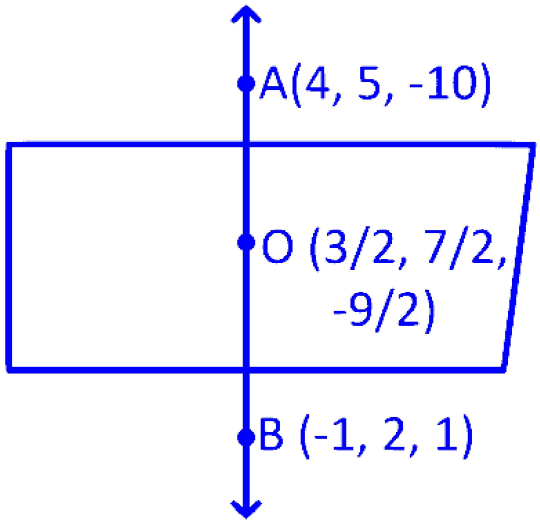
D. $10x + 6y - 22z + 185 = 0$

Answer: B

Solution:



$$\mathbf{AB} = \mathbf{n} = \mathbf{b} - \mathbf{a} = -5\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$$

Plane is,

$\bar{\mathbf{r}} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$

$$\bar{\mathbf{r}} \cdot (-5\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 11\hat{\mathbf{k}}) = -\frac{15}{2} - \frac{21}{2} - \frac{99}{2}$$

$$\Rightarrow -5x - 3y + 11z = -\frac{135}{2}$$

$$\Rightarrow -10x - 6y + 22z + 135 = 0$$

$$\Rightarrow 10x + 6y - 22z - 135 = 0$$

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